Calibration of the Hubble Space Telescope Fine Guidance Sensors: An Application of Seminal Ideas of H.K. Eichhorn

William H. Jefferys

May 11, 1998

Abstract

The calibration of the Hubble Space Telescope Fine Guidance Sensors (FGS) posed a number of unique challenges. There existed no star field of sufficient size known to the precision and accuracy required to calibrate the optical field angle distortion (OFAD) of the telescope. Therefore, it was necessary to use observations by the FGS to calibrate itself. This involved application of a number of ideas that H.K. Eichhorn has developed and used: Overlapping plate models, rigorous use of statistical methods, statistical analysis of the errors introduced by the plate constants, and global solution methods. In this paper I will discuss the many influences that Eichhorn's seminal ideas had on solving this important astrometric problem.

1 Introduction

The Hubble Space Telescope (HST) Fine Guidance Sensors (FGS) were designed to serve two purposes: fine pointing of the telescope and astrometry. The fine-pointing function is an engineering function, and the FGS form an integral part of the telescope's pointing and control system (PCS) together with the gyros, the coarse star sensors, and the reaction wheel assembly. But the second function is the one we are interested in. Because the fine pointing of the telescope was specified to be $\pm 0.007''$, the capability of very precise position measurement had to be built into the FGS units anyway, so special efforts were made to ensure that they could be used for astrometry, at a design level of $\pm 0.0027''$ *RMS*, combined in x and y.

My purpose in this paper is not to go into great detail about the astrometric use of the FGS, but instead to outline the calibrations we had to accomplish for one of the astrometric modes, the so-called POS (positional) mode, with special attention to the ways in which Heinrich Eichhorn influenced our choice of methods and techniques.

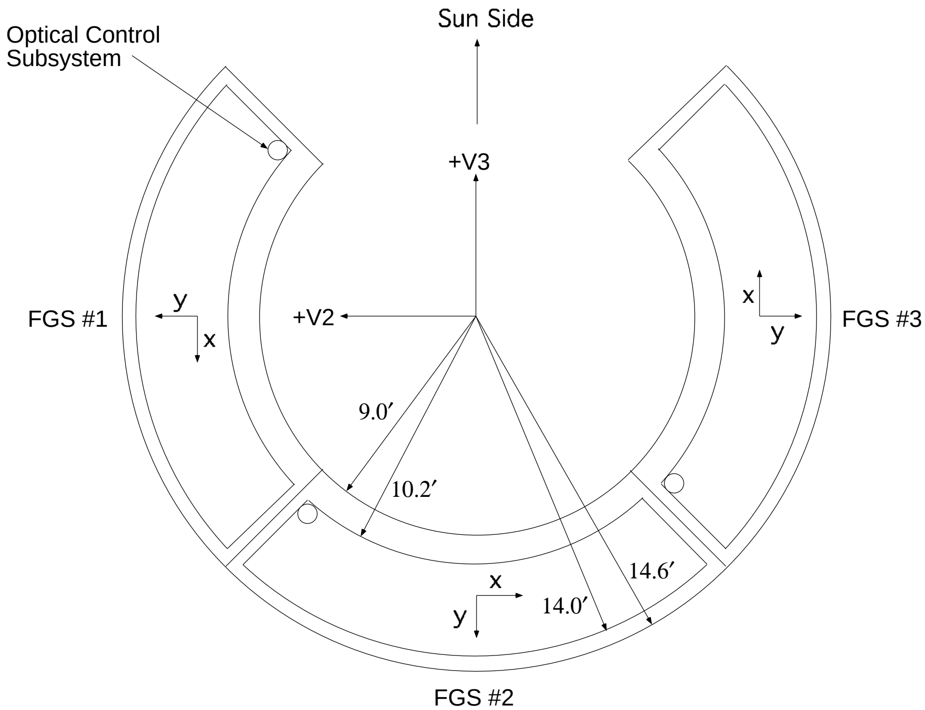

Figure 1: The HST Field of View, showing the location of the FGS

The Hubble Space Telescope is a 2.4-meter, f-24 instrument of Ritchey-Chrètien design. As such it has a large amount of radially symmetric optical distortion. Fortunately, the well-known problem of spherical aberration due to mirror misfigure doesn't affect astrometric *accuracy* although it does reduce fringe visibility in the FGS Koester's prism interferometer. The problem for the FGS is made even more severe because the FGS units are located well away from the optical axis of the telescope (see Figure 1). Long before launch, it was understood that calibration for the optical field angle distortion (OFAD) would be required. In a perfect telescope, the distortion can be modelled as

$$\begin{aligned} x' &= \rho_3 x(x^2+y^2) + \rho_5 x(x^2+y^2)^2 \\ y' &= \rho_3 y(x^2+y^2) + \rho_5 y(x^2+y^2)^2 \end{aligned} \tag{1}$$

However, with the real telescope, additional terms must be included to fully model optical misfigure and misalignment of the many elements in the system, and the full distortion polynomial, with enough terms to recover the astrometry error budget to the level of 1 milliarcsecond (MAS) is of the form [DAR84]

$$
\begin{aligned}
x' &= a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{30}x(x^2+y^2) \\
&\quad + a_{21}x(x^2-y^2) + a_{12}y(y^2-x^2) + a_{03}y(y^2+x^2) \\
&\quad + a_{50}x(x^2+y^2)^2 + a_{41}y(x^2+y^2)^2 + a_{32}x(x^4-y^4) \\
&\quad + a_{23}y(y^4-x^4) + a_{14}x(y^2-x^2)^2 + a_{05}y(y^2-x^2)^2 \\
y' &= b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{11}xy + b_{02}y^2 + b_{30}x(x^2+y^2) \\
&\quad + b_{21}x(x^2-y^2) + b_{12}y(y^2-x^2) + b_{03}y(y^2+x^2) \\
&\quad + b_{50}x(x^2+y^2)^2 + b_{41}y(x^2+y^2)^2 + b_{32}x(x^4-y^4) \\
&\quad + b_{23}y(y^4-x^4) + b_{14}x(y^2-x^2)^2 + b_{05}y(y^2-x^2)^2 \qquad (2)
\end{aligned}
$$

The standard method of evaluating the coefficients of such a polynomial would be to rely upon an already-calibrated field of standard stars. Such a field did not exist to sufficient accuracy to do the calibration this way, and so it became necessary to resort to a different strategy. If the OFAD were properly calibrated, then observing the same field of view at various orientations and positions should yield the same relative positions of the stars. This exploits the metric invariance of the star field under repointings of the telescope, which naturally leads to an overlapping plate problem, as pioneered by Eichhorn [Eic60] (see also [GEL70] and [Jef79]).

Various practical considerations led to the choice of a field in M35 as the calibration field. This field is relatively rich and of the right angular size to match the size of the HST FGS field of view. The stars are of the right magnitude range ($10 \leq V \leq 14$). These characteristics permit a large number of stars to be measured in the relatively short period available during a single orbit. A lower accuracy set of positions [MS86] was already available from groundbased observations, which enabled an “engineering” OFAD calibration adequate for pointing purposes to be conducted alongside the “scientific” calibration using the same set of data. Finally, the field is near the ecliptic, so that there is a time of year during which the field is 180° away from the sun, permitting any orientation of the telescope to be used relative to the cluster.

The final OFAD calibration involved measurements of some 50 stars and 19 pointings of the telescope with various offsets and rolls designed to obtain the maximum leverage on the OFAD coefficients (Figure 2). Later I will discuss some of the techniques we used in settling upon a particular overlap pattern.

2 Formulation of the Model

From the beginning, our goal was to model the geometric optics of the fine guidance sensors as closely as possible, and to leave the Eqs. (2) as the only empirical aspect of the model. The fine guidance sensors are a fairly complex

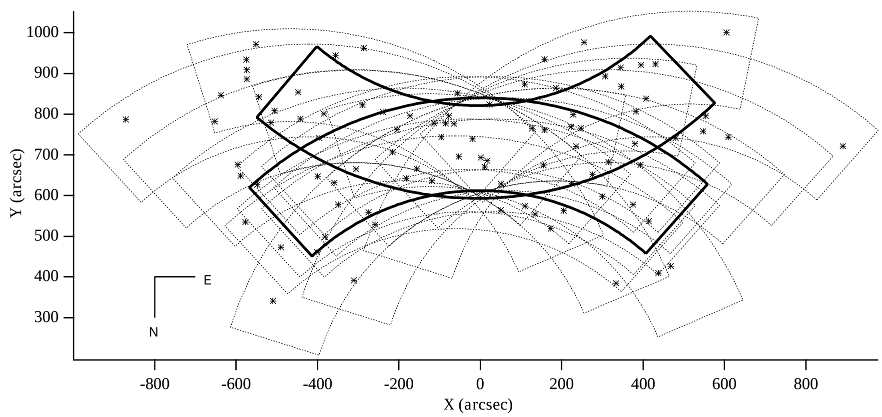

Figure 2: The overlap pattern for the HST OFAD calibrations

optical-mechanical system (Figure 3) including a pick-off mirror, a collimating asphere, and two star selectors that bring the light from a star onto the axis of the detector by rotating the detector axis around the optical axis with an offset of a fixed angle, each by about 7.1° (Figure 4). Stars are observed by moving the instantaneous field of view (FOV) from star to star, integrating on each star for a sufficient time (depending on magnitude) to obtain the desired precision, typically 60 seconds. Some of the optics in the system have power, so the actual field of view of the FGS goes approximately from $10.2'$ to $14'$ from the optical axis of the telescope.

In addition, we decided to model the star positions as unit vectors pointing towards the stars, rather than to use standard coordinates, which would not have been appropriate given the particular optical-mechanical configuration of the FGS. This follows an idea originally suggested by Eichhorn [Eic71], who proposed direct adjustment of the right ascensions and declinations of the stars. In our case, it seemed preferable to work with the unit vectors, though this does introduce a complication in that we need to enforce the constraint that the unit vectors have unit length. On the other hand, it makes it easy to handle the offsets introduced by the overlapping fields. This could be done with matrices, but we elected to accomplish it using unimodular (unit length) quaternions. The quaternion method was published in [Jef87], and it has proven to be very effective. It has the advantage that the equations describing the telescope repointings are purely algebraic and involve no trigonometric functions. They are free of the singularities that would arise if Euler angles were used. With this approach, the statistical model for the OFAD calibration becomes

$$Q_p^{\dagger}\hat{\xi}Q_p = \hat{u} + \Delta\hat{u} + \hat{\epsilon}$$

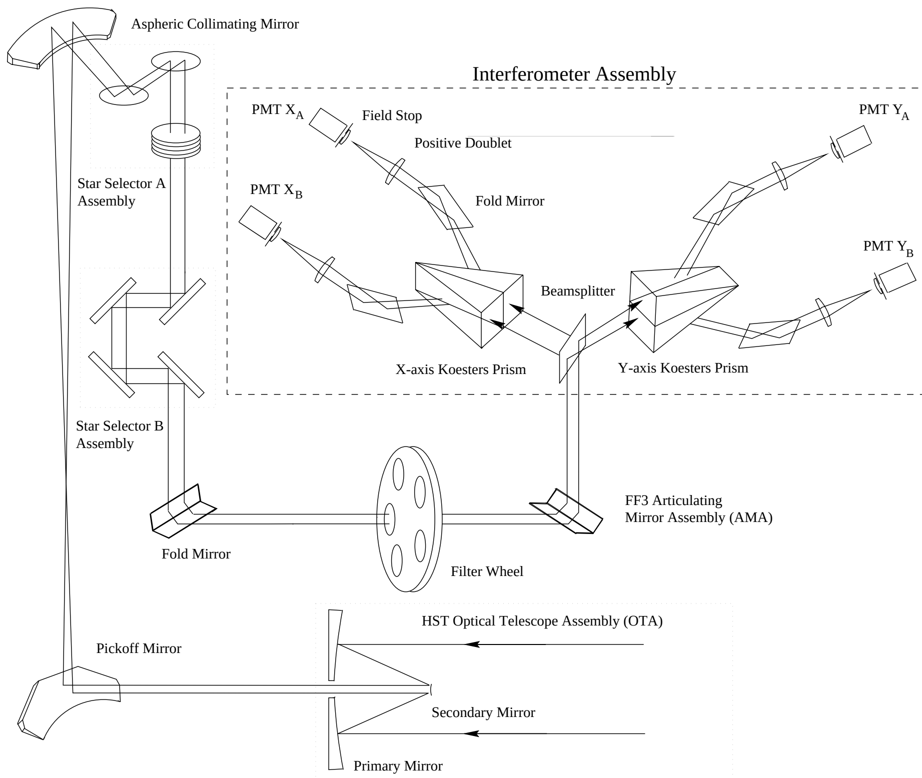

Figure 3: Schematic diagram of the FGS units.

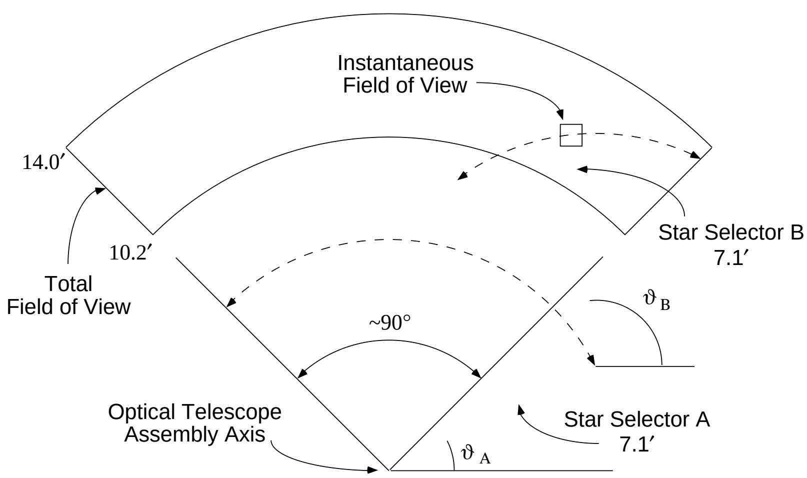

Figure 4: The geometry of the FGS star selectors

with

$$Q_p^\dagger Q_p = 1$$

where $\hat{u}$ is the observed position vector in the FGS *instrumental* frame; $\hat{\xi}$ is the unknown parameter of star positions in a fixed *reference* frame; Q_p is an unknown unimodular quaternion for transforming the different pointings p to a reference pointing; $\hat{\epsilon}$ is the measurement error; and $\Delta\hat{u}$ contains the systematic corrections for all of the components of the model, some of which (such as groundbased measurements of the deviations of pick–off mirror and asphere from "ideal") are already known and some of which involve parameters (such as the constants in Eqs. 2) that are to be determined. In particular, the quantities (x, y) of Eqs. (2) are simply the first and second components of $\hat{u}$. Also included in the model are physical effects such as differential aberration due to spacecraft motion. Details of the rather complex model can be found in [JWW$^+$94].

3 Some Statistical Considerations

Heinrich Eichhorn has been a consistent advocate of the use of rigorous methods of statistical adjustment of data. He early became aware of the power of electronic computers and recognized that this development necessarily entailed a revolution in the way that astrometry (in particular) was being done. He pointed out that the existence of computers meant that there was no longer any excuse to rely upon approximate methods of reduction, and that only rigorous methods would suffice in the future. He rediscovered and taught the use of the methods of Deming [Dem38] and Brown [Bro55], and has also published important refinements of these methods [ER76]. Inspired by his example and encouragement, I also sought to understand and improve upon Deming's and Brown's methods, which resulted in several papers ([Jef80], [Jef81], [Jef90]), in which a rigorously correct treatment of the fully nonlinear maximum likelihood estimator is developed, and then extended to the robust regime to account for possible outliers in the data.

This work also led us to write the computer program GaussFit [JFM88], which embodies in software the ideas in these papers. GaussFit is our attempt to make it very easy to apply rigorous statistical reduction algorithms, especially to astrometric data. It includes a computer language that is specifically designed for expressing complex least-squares problems at the highest level. Essentially, one writes an algorithm that embodies the *physical/mathematical model* that one is trying to solve in a very high-level language; GaussFit's compiler translates this into a program for a virtual computing machine that emulates that model. The virtual computing machine knows about derivatives and keeps track of them automatically and analytically in all calculations. It is therefore able to compute the matrices required for the algorithms exactly. The matrix equations are solved using a computationally efficient QR decomposition method that has good properties of numerical stability.

GaussFit turned out to be essential for our actual application to this problem. Since we wanted to use an accurate physical model of the FGS, the actual mathematical representation of the model is rather complex, and it would have been orders of magnitude more difficult for us to implement the derivative part of the code "by hand." But, since GaussFit takes care of the derivatives automatically, we did not have to cope with that aspect of the problem and were able to concentrate on making sure that the mathematical model was correct. Furthermore, GaussFit gave us great freedom in trying out various approaches to our problem.

An essential feature of Deming's approach to least squares, which is inherited in all of the subsequent work I cited, is the ability to enforce exact constraints. We programmed this capability into GaussFit. It is needed in our application, for example, to enforce the constraints that the position vectors representing the star positions are unit vectors, and that the quaternions that represent the different pointings of the spacecraft are unimodular. With GaussFit, each of these constraints can be enforced for all cases with a single line of GaussFit code.

The ability to enforce constraints turned out to be crucial in another way. The nature of the problem at hand turns out to include a number of *unidentified parameters*, which cannot be determined uniquely from the data and must therefore be constrained in order to obtain an adequate solution. For example, one pointing of the telescope is designated as a *reference pointing*, and all other pointings are referred to it. The unit vectors $\hat{\xi}$ are also referred to this pointing. Therefore, we need to constrain the quaternion of the first pointing to represent no rotation at all. Similarly, we must enforce constraints associated with scale changes ("gauge constraints"). The very simple, elegant tools available in GaussFit made it easy for us to include such constraints and to experiment with the best way to formulate them.

4 Choice of Overlap Pattern

In choosing the overlap pattern (Figure 2) we were faced with several operational constraints: First, the amount of spacecraft time that would be allowed for the calibration would be limited. HST time is precious, and although this is a crucial calibration both for engineering and for scientific (astrometric) purposes, it was our responsibility to design a test that would get adequate accuracy with a minimum expenditure of spacecraft resources. Second, it was necessary to take all of the data for a single pointing of the spacecraft during a single orbit, to maintain the homogeneity of the data from that pointing. And third, we had to remeasure several "check stars" during the course of the measurement to guard against unexpected events that might compromise the data take. Early in the mission, for example, "Hubblequakes" resulting from sudden releases of energy due to "stiction" in the solar arrays were a major problem. Another serious problem—for astrometry—was the changing characteristics of the PCS and the FGS, due to thermal effects during the course of the observation.

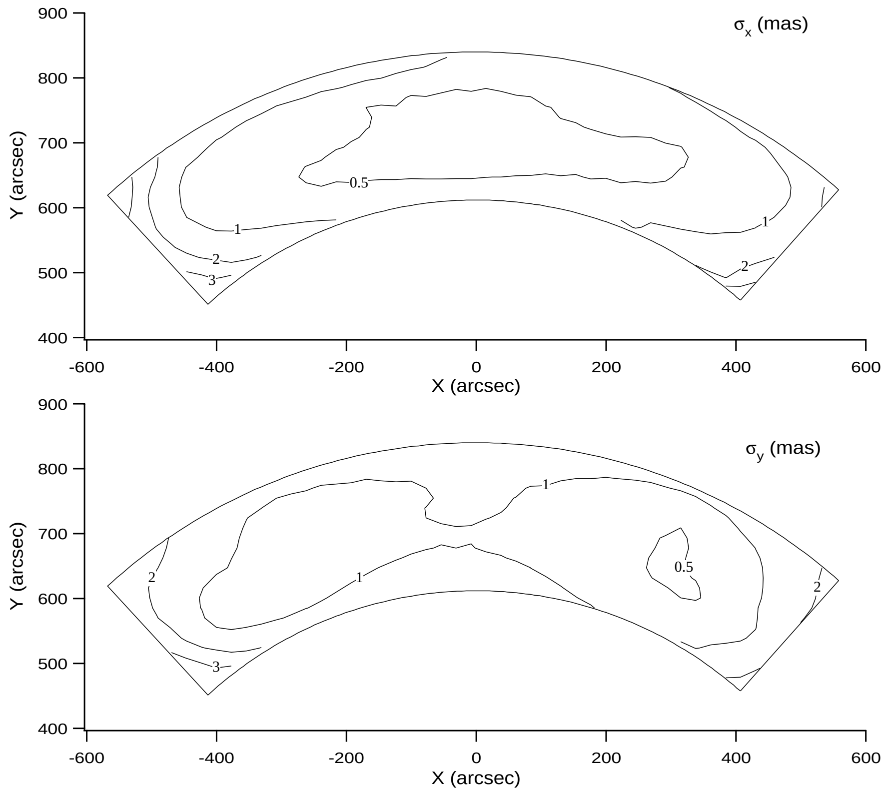

Figure 5: Bootstrap predictions of astrometric accuracy over the FGS field of view

For these reasons we were limited to about 30 different stars per pointing. Naturally we tried to devise overlap patterns that would maximize the number of times a given star would appear in different pointings, since that is the source of the information that allows us to learn about the field angle distortions. But it was also necessary to understand how different pointing patterns would affect the statistical information obtained. For this reason, we resolved to use ideas suggested by Eichhorn and Williams [EW63] to understand the variation of the error introduced by our adjustment as a function of position within the FGS field of view. Other work on this approach can be found in [vdH77].

Eichhorn and Williams used analytical inversion of the matrix of the normal equations of a plate adjustment to compute the error introduced by the adjustment as a function of position on the plate. They therefore assumed a smooth distribution of stars and approximated the product sums by integrals. In our case, such a procedure would have been unfeasable because of the very large size of the overlapping plate problem (with approximately 200 unknowns and 1200 equations of condition). It also would have been undesirable since it would have thrown away information that the actual distribution of the stars had on the solution, since we had already settled upon a particular field of stars.

Therefore we looked for a method that would enable us to obtain equivalent information by simulation. We decided to use a statistical technique known as "bootstrapping" [Efr82] to provide the needed information. With this technique, one simulates the information on the statistical problem with a Monte Carlo technique. This is done by making a baseline solution with the original data (this can actually be observed data, however in the simulations we did in the planning stages we used artificial data), and then creating a "library" of residuals for that solution. By sampling with replacement from the "library," we can create a large number of data sets out of the reference solution which have the same statistical properties as the original data set. Each data set is then solved using the same GaussFit program that we developed to analyze the OFAD data. The analysis produces a simulated statistical sample of errors for each of the stars in the solution. These in turn can be analyzed to inform us about the properties of the errors as a function of position in the field of view (Figure 5). This work was done by my student Q. Wang [Wan90] for his master's thesis.

A nineteen orbit calibration (Figure 2) was performed in the spring of 1993 for the initial on-orbit calibration of the optical field angle distortion. Additional partial calibrations have been performed alongside regular stability tests. These have allowed us to maintain an error level of better than 2 milliarcseconds over most of the field of view of the FGS.

5 Summary

Our survey of the calibration of the Hubble Space Telescope fine guidance sensors has been of necessity brief. We have not touched upon the calibration of the transfer function, used for the observation of close ($\geq$ 10 MAS) double stars, which is a story in its own right. We have deliberately obscured the complexities of the model for POS mode astrometry, and have not mentioned refinements in the model that we had to add to compensate for unmodelled errors that were discovered as a result of a careful investigation of the residuals of the OFAD calibration. Nonetheless, we have shown in this paper how profound the influence of Heinrich Eichhorn has been on just this one problem. It is only one example of the influence that his career has had on all of astrometric science. On this occasion of his $70th$ birthday, we thank him for all that he has contributed to our endeavors.

6 Acknowledgements

I thank the National Aeronautics and Space Administration for their generous support through NASA Contract NAS8-32906 and NASA Grant NAG5-1603 which supported this work. I am grateful to my many colleagues, too numerous to mention individually, who developed the methods and tools we used in this project and applied them to this crucial task.

References

[Bro55] Duane Brown. A matrix treatment of the general problem of least squares considering correlated observations. Technical Report Report No. 937, Ballistic Research Laboratories, 1955.

[DAR84] G. Dente and L. Abramowicz-Reed. Space Telescope optical field angle distortion calibration algorithm. Technical Report ST-SE-3360, Perkin-Elmer, 1984.

[Dem38] W.E. Deming. *Statistical Adjustment of Data*. John Wiley & Sons, New York, 1938.

[Efr82] Bradley Efron. *The Jackknife, the Bootstrap and Other Resampling Schemes*. Society for Industrial and Applied Mathematics, Philadelphia, 1982.

[Eic60] Heinrich Eichhorn. Über die Reduktion von photographischen Sternpositionen und Eigenbewegungen. *Astron. Nachr.*, 285:233–237, 1960.

[Eic71] Heinrich Eichhorn. On the direct use of spherical coordinates in photographic astrometry. *Annalen Universitäts—Sternwarte Wien*, 30(1):5–13, 1971.

[ER76] Heinrich Eichhorn and Jane Russell. An algorithm for the statistically rigorous and non-interative computation of trigonometric parallaxes. *Mon. Not. R. Astr. Soc.*, 174:679–693, 1976.

[EW63] Heinrich Eichhorn and Carol Ann Williams. On the systematic accuracy of photographic astrometic data. *Astron. J.*, 68:221–231, 1963.

[GEL70] William D. Googe, Heinrich Eichhorn, and Carl F. Lukac. The overlap algorithm for the reduction of photographic star catalogues. *Mon. Not. R. Astr. Soc.*, 150:35–44, 1970.

[Jef79] William H. Jefferys. On the overlapping plate method in astrometry. *Astron. J.*, 84:1775–1777, 1979.

[Jef80] William H. Jefferys. On the method of least squares. *Astron. J.*, 85:177–181, 1980.

[Jef81] William H. Jefferys. On the method of least squares. II. *Astron. J.*, 86:149–155, 1981.

[Jef87] William H. Jefferys. Quaternions as astrometric plate constants. *Astron. J.*, 93:755–759, 1987.

[Jef90] William H. Jefferys. Robust estimation when more than one variable per equation of condition has error. *Biometrika*, 77:597–607, 1990.

[JFM88] W.H. Jefferys, M.J. Fitzpatrick, and B.E. McArthur. GaussFit—a system for least squares and robust estimation. *Celestial Mechanics*, 41:39–49, 1988.

[JWW+94] W. Jefferys, A. Whipple, Q. Wang, B. McArthur, G.F. Benedict, E. Nelan, D. Story, and L. Abramowicz-Reed. Optical field angle distortion calibration of FGS3. In J. Chris Blades and Samantha J. Osmer, editors, *Calibrating Hubble Space Telescope*, pages 353–373. Space Telescope Science Institute, Baltimore, MD, 1994.

[MS86] B. McNamara and K. Sekiguchi. *Astronomical Journal*, 91:557, 1986.

[vdH77] K. von der Heide. Accuracy of the average of star positions obtained from overlapping plates. *Astron. Astrophys.*, 61:545–552, 1977.

[Wan90] Qiangguo Wang. Statistical analysis of the optical field angle distortion of the Hubble Space Telescope using the bootstrap method. Master's thesis, University of Texas at Austin, 1990.